

LAST NAME: _____

FIRST NAME: _____

Problem 1 (a) Calculate the image of the sequence $\langle 3, 0, 2 \rangle$ under Gödel numbering and show your work. If this image does not exist, prove it.

Answer:

$$\begin{aligned}
 & 2^{3+1} \cdot 3^{0+1} \cdot 5^{2+1} = \\
 & 2^4 \cdot 3 \cdot 5^3 = 10^3 \cdot 6 = \\
 & \boxed{16000}
 \end{aligned}$$

(b) Calculate the pre-image of the number 2940 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

Answer:

$$\begin{aligned}
 2940 &= 10 \cdot 294 = \\
 & 10 \cdot 3 \cdot 98 = \\
 & 2 \cdot 3 \cdot 5 \cdot 2 \cdot 49 = \\
 & 2^2 \cdot 3 \cdot 5 \cdot 7^2
 \end{aligned}$$

answer:

$$\boxed{\langle 1, 0, 0, 1 \rangle}$$

(c) Calculate the pre-image of the number 3850 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

Answer:

$$3850 = 385 \cdot 10 =$$

$$2 \cdot 5 \cdot 7 \cdot 55 =$$

$$2 \cdot 5^2 \cdot 7 \cdot 11$$

not a Gödel number,

misses 3, no

pre-image.

(d) Let m be the image of the sequence

$$s = \langle x_1, x_2, x_3, x_4, x_5, x_6 \rangle$$

under Gödel numbering. Represent the pre-image of the number $78m$ as a function of the (components of) sequence s . If such a representation does not exist, prove it.

Answer:

$$\begin{aligned}
 78m &= 6 \cdot 13 = \\
 &= 2 \cdot 3 \cdot 13
 \end{aligned}$$

answer:

$$\boxed{\langle x_1+1, x_2+1, x_3, x_4, x_5, x_6+1 \rangle}$$

(e) Let n be the image of the sequence

$$\langle x_1, x_2, x_3, x_4 \rangle$$

under Gödel numbering. Represent the image of the sequence

$$\langle x_1+2, x_2+1, x_3, x_4, 1 \rangle$$

as a function of n . If such a representation does not exist, prove it.

Answer:

answer

$$n \cdot 2^2 \cdot 3 \cdot 11^2 =$$

$$= 12 \cdot 121n$$

$$\boxed{= 1452n}$$

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Problem 1 (a) Calculate the image of the sequence $\langle 5, 0, 1 \rangle$ under Gödel numbering and show your work. If this image does not exist, prove it.

Answer:

$$2^{5+1} \cdot 3^{0+1} \cdot 5^{1+1} = 2^6 \cdot 3 \cdot 5^2 = 10^2 \cdot 3 \cdot 10 = 14800$$

(b) Calculate the pre-image of the number 2730 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

Answer:

$$2730 = 10 \cdot 273 = 2 \cdot 5 \cdot 3 \cdot 91 = 2 \cdot 3 \cdot 7 \cdot 13$$

misses 11, not a Gödel number, no pre-image

(c) Calculate the pre-image of the number 6930 under Gödel numbering and show your work. If this pre-image does not exist, prove it.

Answer:

$$6930 = 10 \cdot 693 = 10 \cdot 9 \cdot 77 = 2 \cdot 5 \cdot 3^2 \cdot 7 \cdot 11 = 2^1 \cdot 3^2 \cdot 5^1 \cdot 7^1 \cdot 11^1$$

answer: $\langle 0, 1, 0, 0, 0 \rangle$

(d) Let \tilde{n} be the image of the sequence

$$\langle x_1, x_2, x_3, x_4 \rangle$$

under Gödel numbering. Represent the image of the sequence

$$\langle x_1+1, x_2, x_3+1, x_4, 2 \rangle$$

as a function of n . If such a representation does not exist, prove it.

Answer:

$$\begin{aligned} n &= 2 \cdot 5 \cdot 11^3 \\ &= 10n \cdot 11 \cdot 121 \\ &= 10n \cdot 1331 \\ &= 13310n \end{aligned}$$

(e) Let m be the image of the sequence

$$s = \langle x_1, x_2, x_3, x_4, x_5, x_6 \rangle$$

under Gödel numbering. Represent the pre-image of the number $143m$ as a function of the (components of) sequence s . If such a representation does not exist, prove it.

Answer:

$$143 = 11 \cdot 13$$

answer:

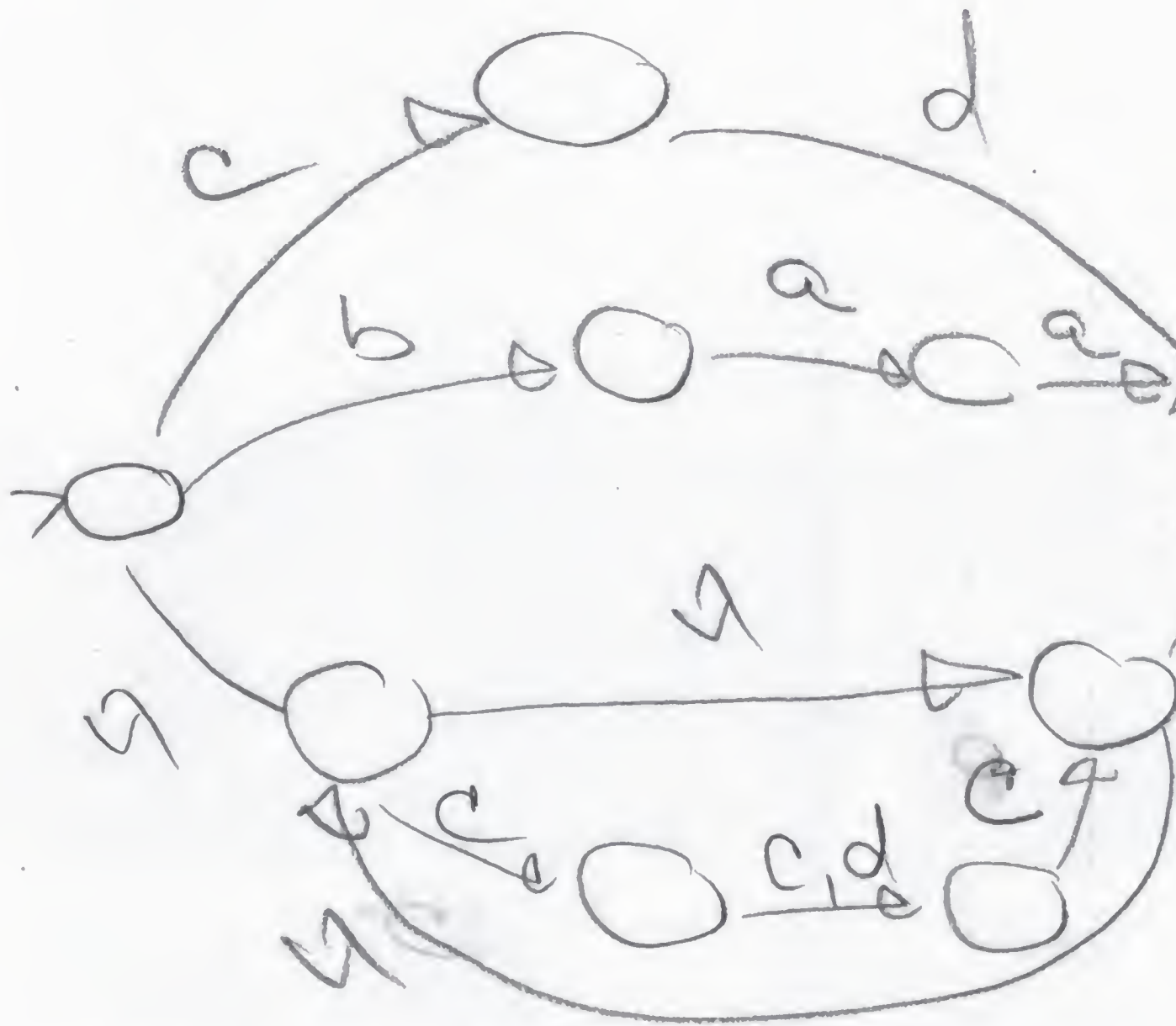
$$\langle x_1, x_2, x_3, x_4, x_5+1, x_6+1 \rangle$$

Problem 2 Let L be the language defined by the regular expression:

$$(cd \cup baa \cup (c(c \cup d)c)^*) (c(da)^* \cup bd^*a)^*$$

(a) Draw a state-transition graph of a finite automaton that accepts the language L . If such an automaton does not exist, state it and explain why.

Answer:



(b) Write a complete formal definition of a context-free grammar that generates the language L . If such a grammar does not exist, state it and explain why.

Answer:

$$G = (V, \Sigma, P, S)$$

$$V = \{S, A, D, B, E, F\}, \Sigma = \{a, b, c, d\}$$

$$\begin{aligned} P: & S \rightarrow AP \\ & A \rightarrow cd \mid baa \mid D \\ & D \rightarrow \epsilon \mid DD \mid ccc \mid cdc \\ & B \rightarrow \epsilon \mid BB \mid cE \mid bFa \\ & E \rightarrow \epsilon \mid daE \mid \epsilon \\ & F \rightarrow \epsilon \mid bF \mid \epsilon \end{aligned}$$

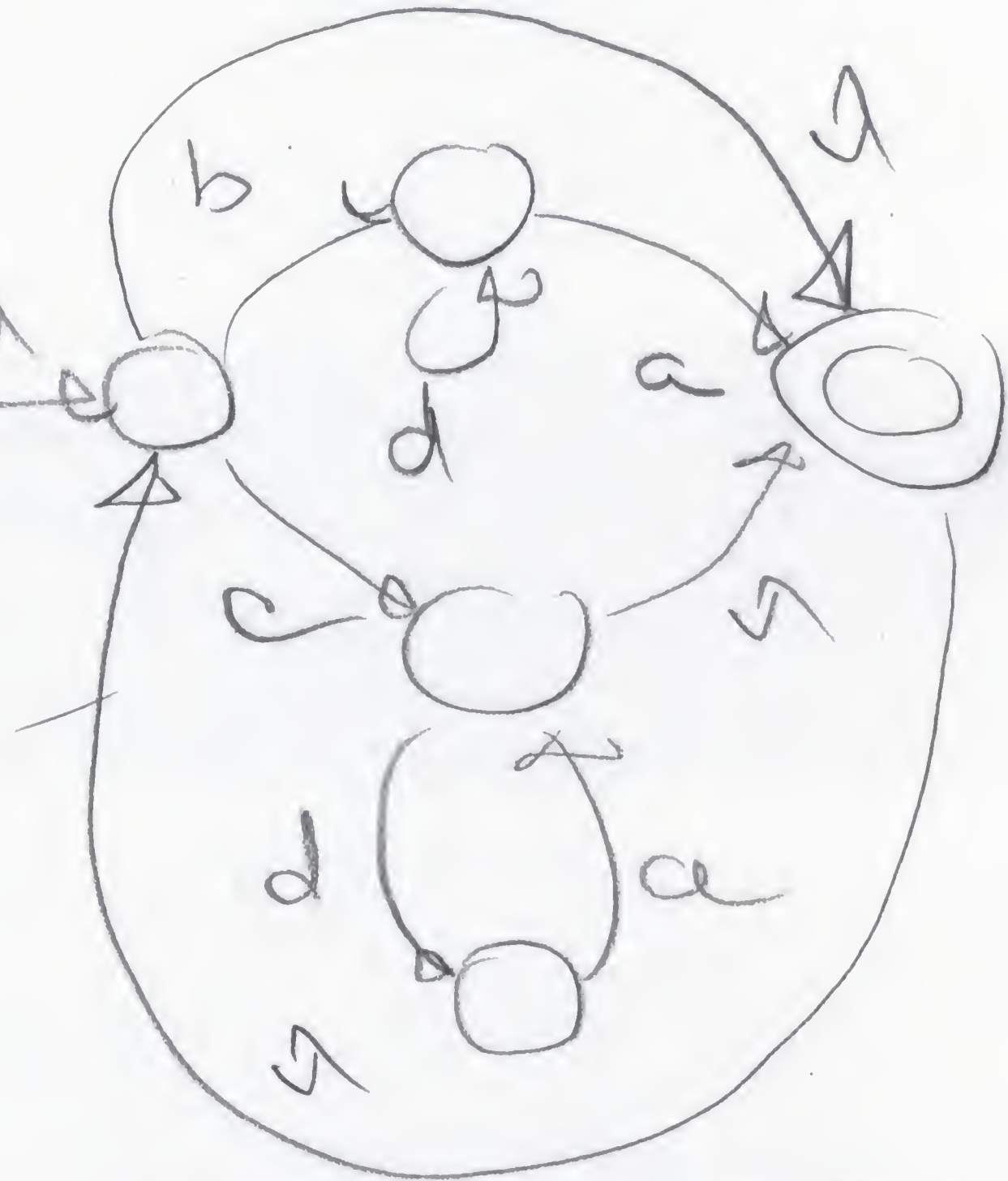
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(c) State the cardinality of the set L . (If L is a finite set, state the exact number of elements of L . Otherwise, state that L is infinite and specify whether it is countable or not.)

Answer:

L is infinite and countable.



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Problem 2 Let L be the language defined by the regular expression:

$$(a(b \cup c)^* \cup db^*c)^* (ab \cup dcc \cup (abaa)^*)$$

(a) Write a complete formal definition of a context-free grammar that generates the language L . If such a grammar does not exist, state it and explain why.

Answer:

$$G = (V, \Sigma, P, S)$$

$$V = \{S, A, B, D, E, H\}$$

$$\Sigma = \{a, b, c, d\}$$

$$P: S \rightarrow AB$$

$$A \rightarrow \epsilon \mid AA \mid aD \mid dEc$$

$$D \rightarrow \epsilon \mid DD \mid b \mid c$$

$$E \rightarrow bE \mid \epsilon$$

$$B \rightarrow ab \mid dcc \mid \epsilon$$

(c) State the cardinality of the set L . (If L is a finite set, state the exact number of elements of L . Otherwise, state that L is infinite and specify whether it is countable or not.)

Answer:

L is infinite and countable

$H \rightarrow \epsilon \mid HH \mid abac$

(b) Draw a state-transition graph of a finite automaton that accepts the language L . If such an automaton does not exist, state it and explain why.

Answer:



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Problem 3 Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c\}$ whose length is not greater than 3.

Let L_2 be the set of exactly those strings over the alphabet $\{a, b, c\}$ where the number of a 's is not less than 2.

(a) Write a regular expression that represents the language L_1 . If such a regular expression does not exist, state it and explain why.

Answer:

$(a|b|c|ab|ac|bc|abc|a|b|c)$

(b) Write a regular expression that represents the language L_2 . If such a regular expression does not exist, state it and explain why.

Answer:

$(b|c)^*a(b|c)^*a(a|b|c)^*$

(c) Write a regular expression that represents the language $L_1 \cup L_2$. If such a regular expression does not exist, state it and explain why.

Answer:

$(a|b|c|ab|ac|bc|abc|a|b|c) \cup (b|c)^*a(b|c)^*a(a|b|c)^*$

(d) Write a regular expression that represents the language $L_1 L_1$. If such a regular expression does not exist, state it and explain why.

Answer:

$(a|b|c|ab|ac|bc|abc)(a|b|c|ab|ac|bc|abc)$

(e) State the cardinality of the set L_1 . (If L_1 is finite set, state the exact number of elements of L_1 . Otherwise, state that L_1 is infinite and specify whether it is countable or not.)

Answer:

$$1 + 3 + 9 + 27 = \boxed{40}$$

(f) State the cardinality of the set L_2 . (If L_2 is a finite set, state the exact number of elements of L_2 . Otherwise, state that L_2 is infinite and specify whether it is countable or not.)

Answer:

infinite and countable

(g) State the cardinality of the set L_1^* . (If L_1^* is a finite set, state the exact number of elements of L_1^* . Otherwise, state that L_1^* is infinite and specify whether it is countable or not.)

Answer:

infinite and countable

(h) State the cardinality of the set $\mathcal{P}(L_2)$ (set of subsets of L_2). (If $\mathcal{P}(L_2)$ is a finite set, state the exact number of its elements. Otherwise, state that $\mathcal{P}(L_2)$ is infinite and specify whether it is countable or not.)

Answer:

infinite and uncountable

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Problem 3 Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c\}$ whose length is equal to 3 or 4.

Let L_2 be the set of exactly those strings over the alphabet $\{a, b, c\}$ where the number of c 's is not less than 3.

(a) Write a regular expression that represents the language L_1 . If such a regular expression does not exist, state it and explain why.

Answer:

$(abc)(abc)(abc)(abc \cup \Lambda)$

(b) Write a regular expression that represents the language L_2 . If such a regular expression does not exist, state it and explain why.

Answer:

$(ab)^*c(ab)^*c(ab)^*c(abc)^*$

(c) Write a regular expression that represents the language L_1L_1 . If such a regular expression does not exist, state it and explain why.

Answer:

$(abc)(abc)(abc)(abc \cup \Lambda)(abc)(abc)(abc \cup \Lambda)(abc \cup \Lambda)$

(d) Write a regular expression that represents the language $L_1 \cup L_2$. If such a regular expression does not exist, state it and explain why.

Answer:

$(abc)(abc)(abc)(abc \cup \Lambda) \cup (ab)^*c(ab)^*c(ab)^*c(abc)^*$

(e) State the cardinality of the set L_1 . (If L_1 is finite set, state the exact number of elements of L_1 . Otherwise, state that L_1 is infinite and specify whether it is countable or not.)

Answer:

$$3^3 + 3^4 = 27 + 81 = 108$$

(f) State the cardinality of the set L_2 . (If L_2 is a finite set, state the exact number of elements of L_2 . Otherwise, state that L_2 is infinite and specify whether it is countable or not.)

Answer:

infinite and countable

(g) State the cardinality of the set $\mathcal{P}(L_2)$ (set of subsets of L_2). (If $\mathcal{P}(L_2)$ is a finite set, state the exact number of its elements. Otherwise, state that $\mathcal{P}(L_2)$ is infinite and specify whether it is countable or not.)

Answer:

infinite and uncountable

(h) State the cardinality of the set L_1^* . (If L_1^* is a finite set, state the exact number of elements of L_1^* . Otherwise, state that L_1^* is infinite and specify whether it is countable or not.)

Answer:

infinite and countable

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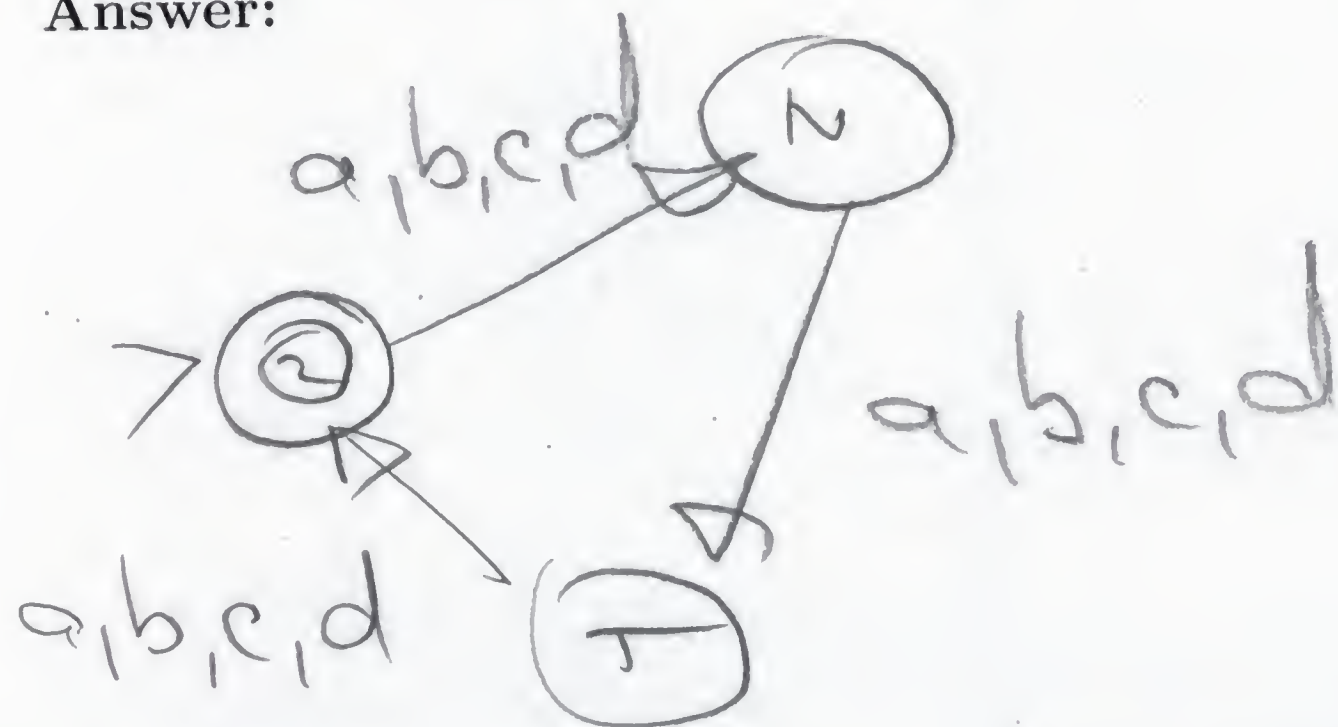
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Problem 4 Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ whose length is divisible by 3.

Let L_2 be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ where the total number of c 's and d 's (together) is odd.

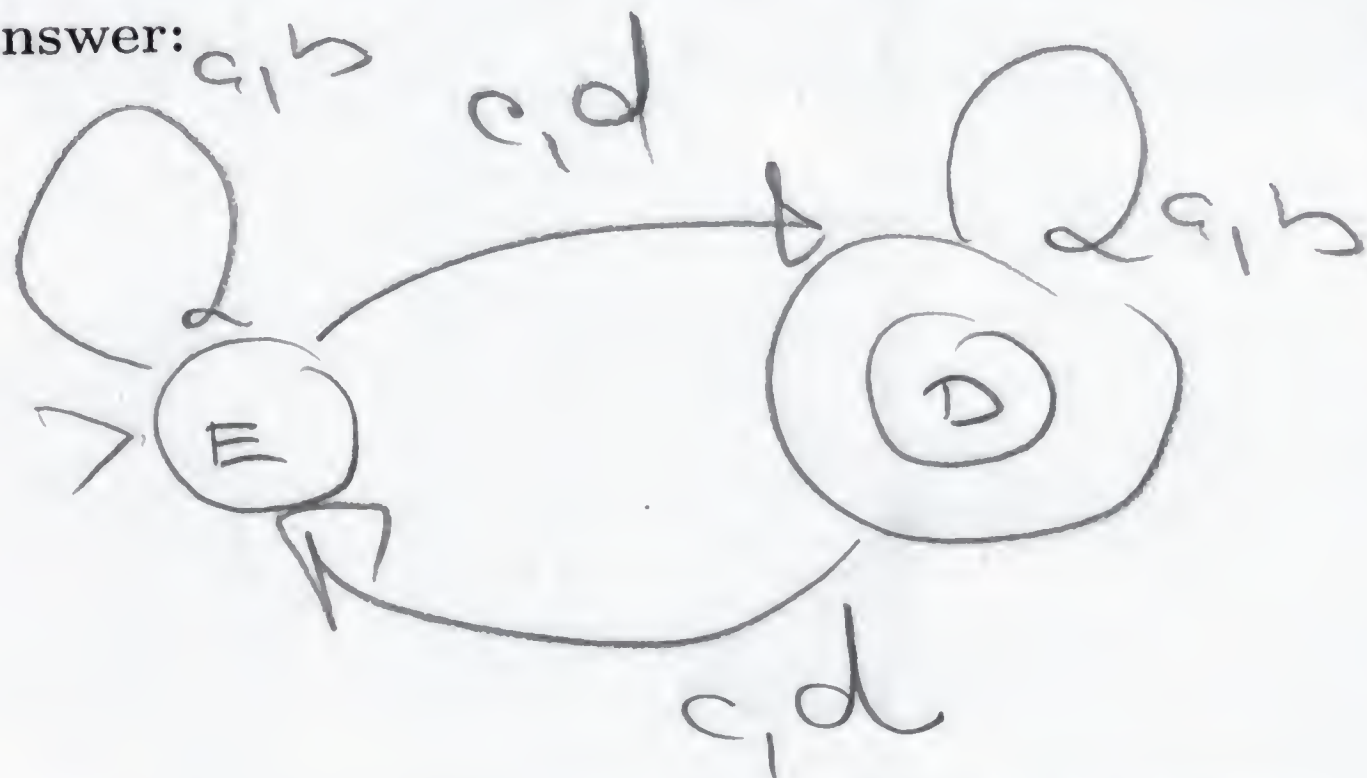
(a) Draw a state-transition graph of a finite automaton that accepts the language L_1 . If such an automaton does not exist, state it and explain why.

Answer:



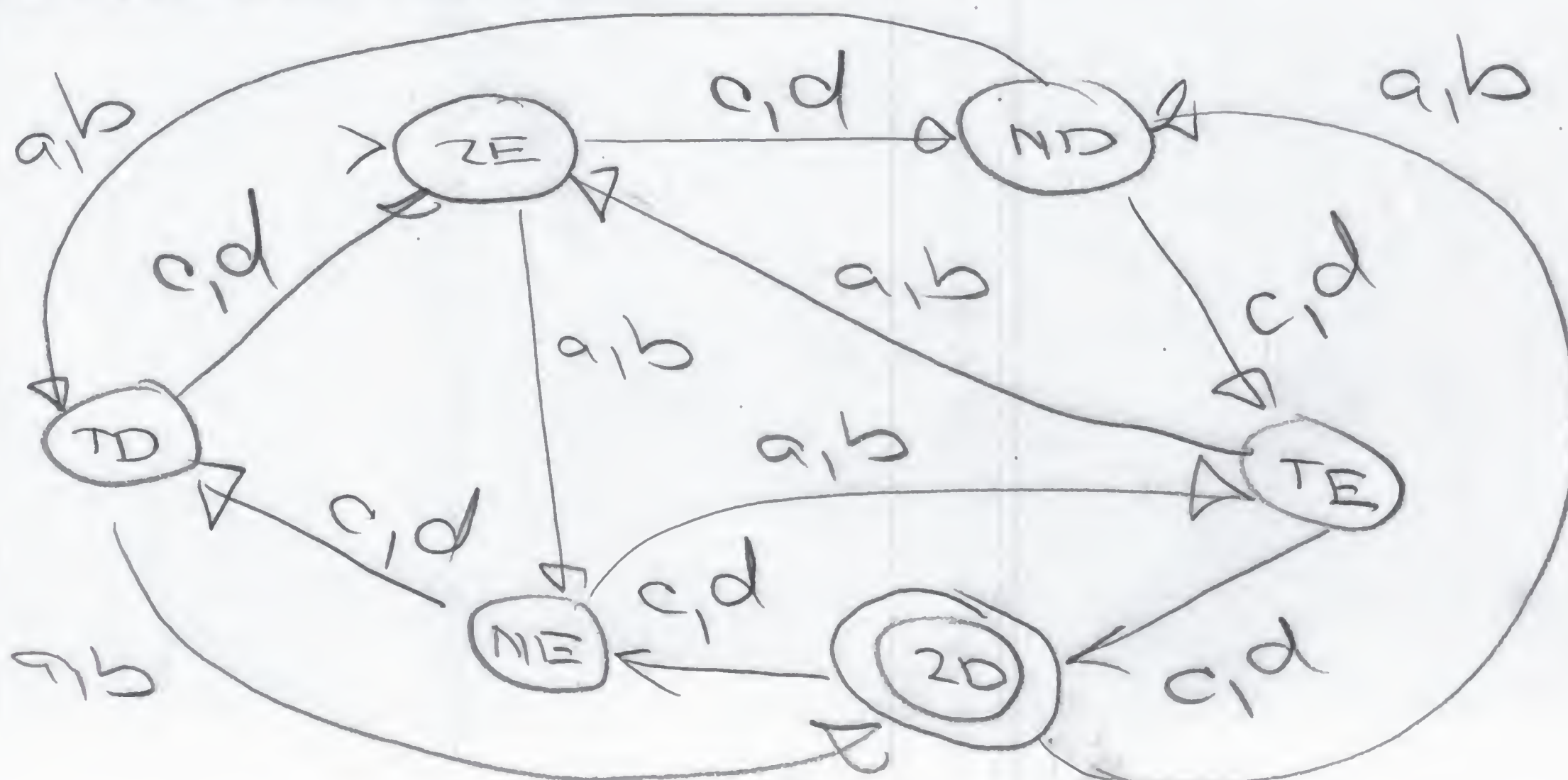
(b) Draw a state-transition graph of a finite automaton that accepts the language L_2 . If such an automaton does not exist, state it and explain why.

Answer:



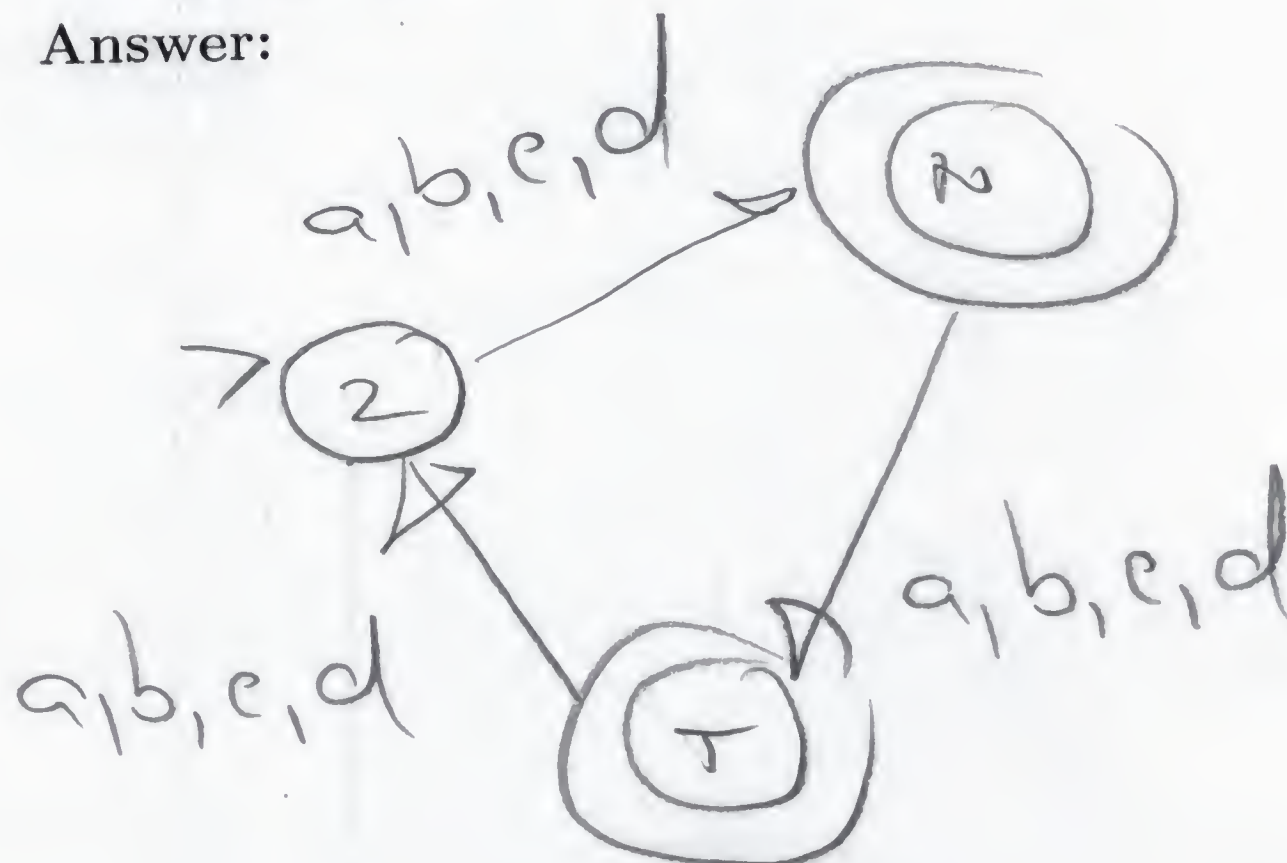
(c) Draw a state-transition graph of a finite automaton that accepts the language $L_1 \cap L_2$. If such an automaton does not exist, state it and explain why.

Answer:



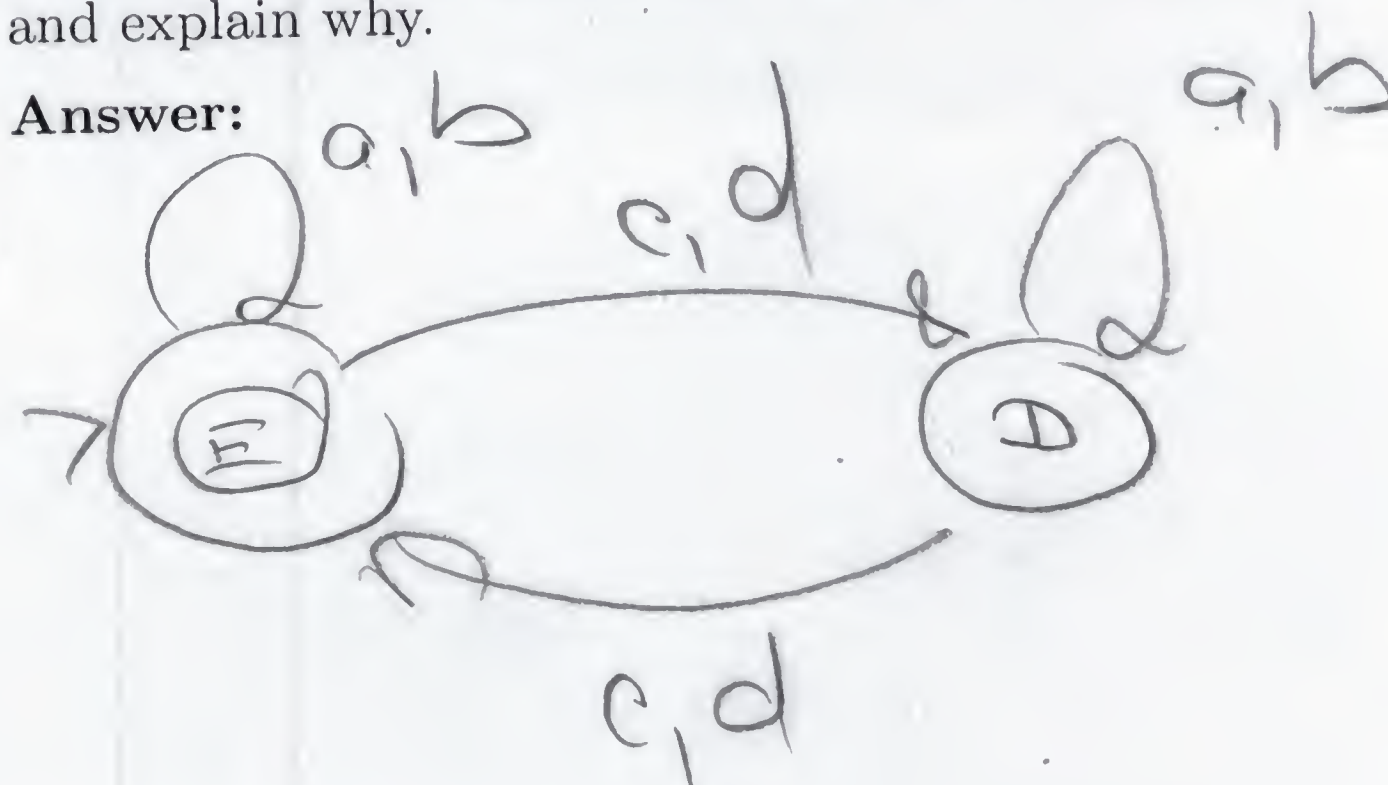
(d) Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_1}$ (the complement of L_1). If such an automaton does not exist, state it and explain why.

Answer:



(e) Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_2}$ (the complement of L_2). If such an automaton does not exist, state it and explain why.

Answer:



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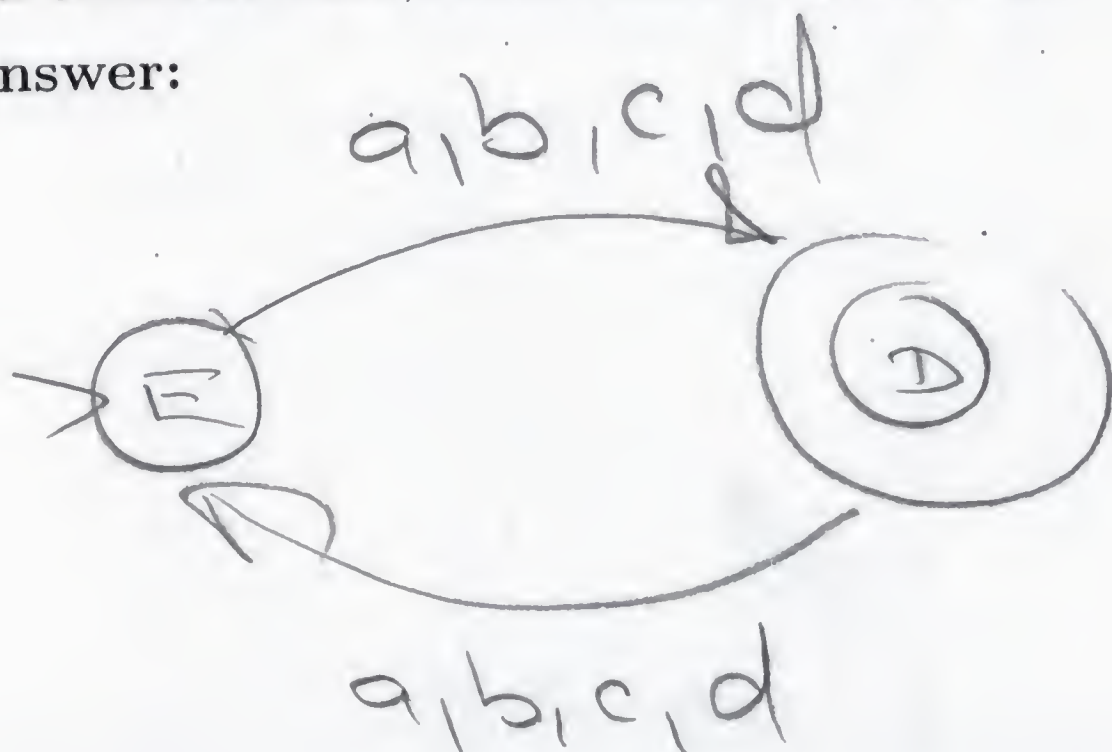
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Problem 4 Let L_1 be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ whose length is odd.

Let L_2 be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ where the total number of a 's and c 's (together) is divisible by 3.

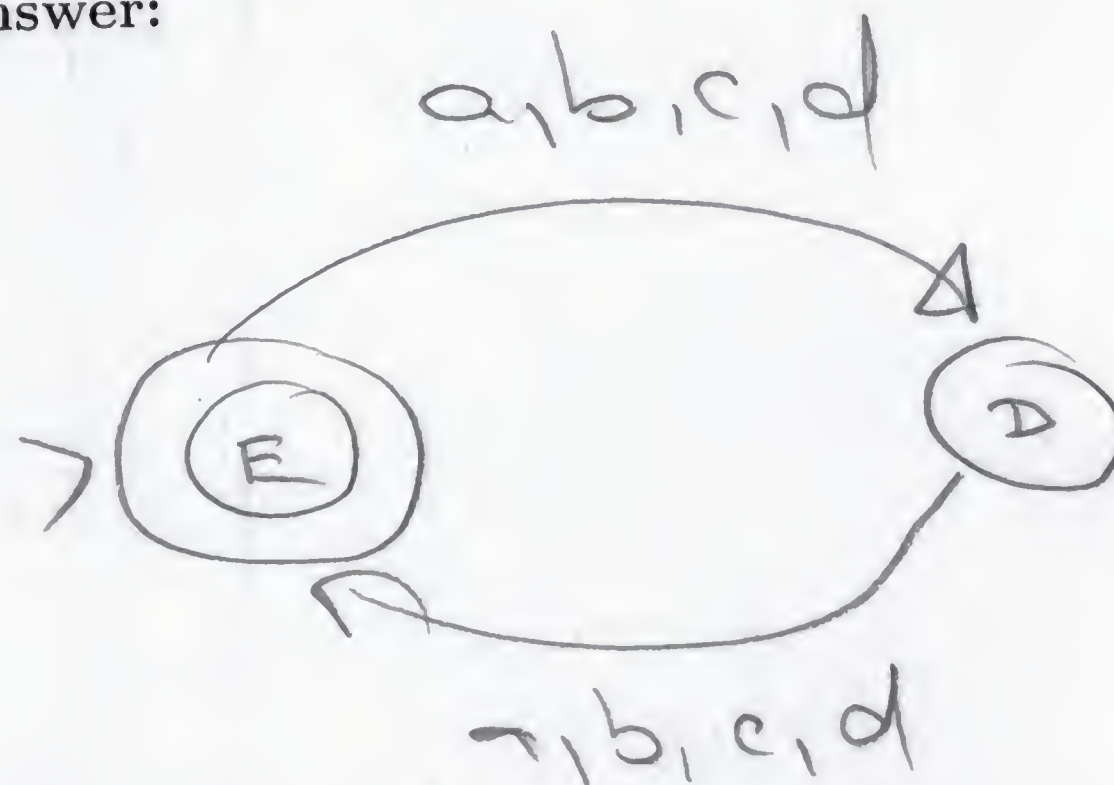
(a) Draw a state-transition graph of a finite automaton that accepts the language L_1 . If such an automaton does not exist, state it and explain why.

Answer:



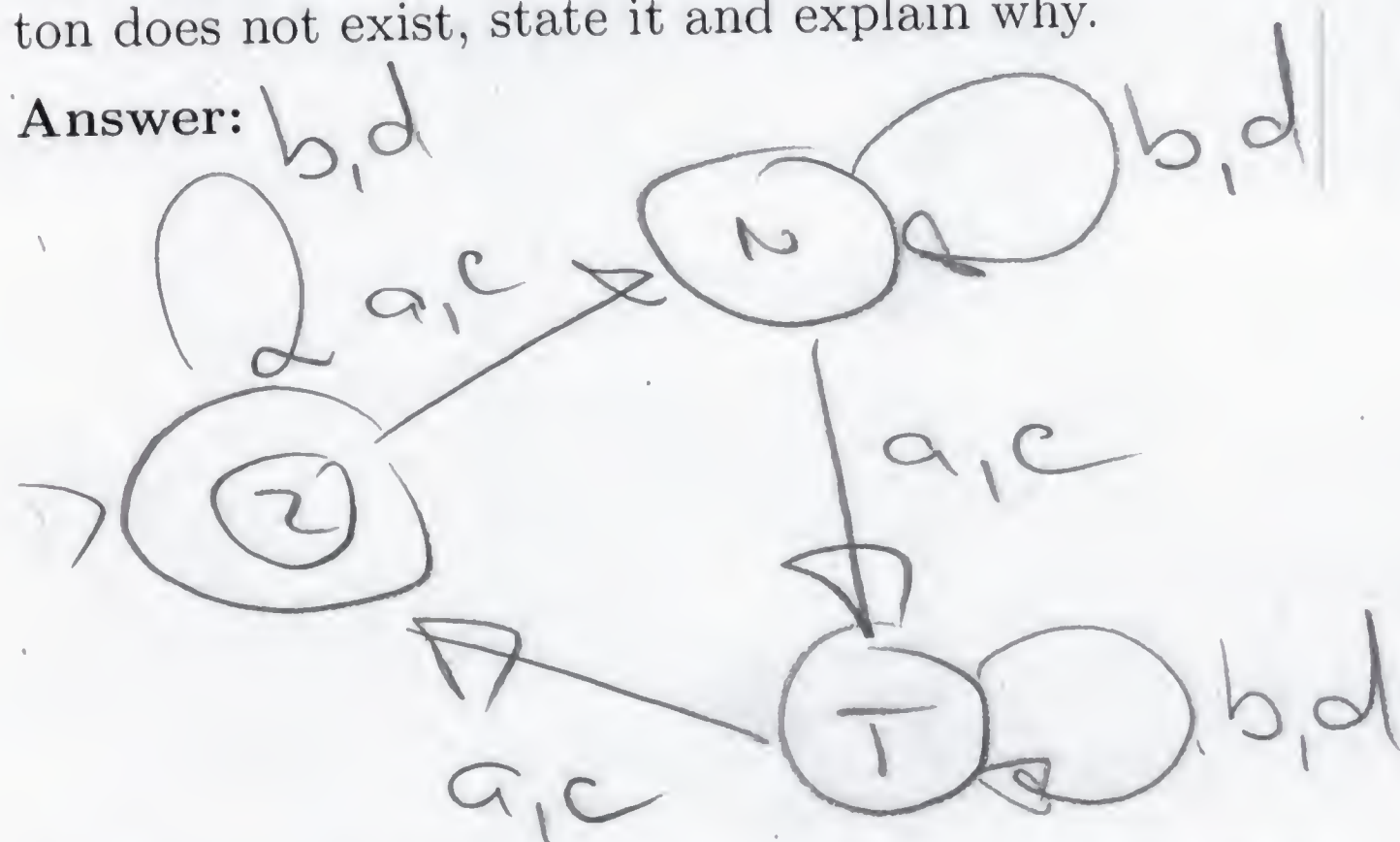
(d) Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_1}$ (the complement of L_1 .) If such an automaton does not exist, state it and explain why.

Answer:



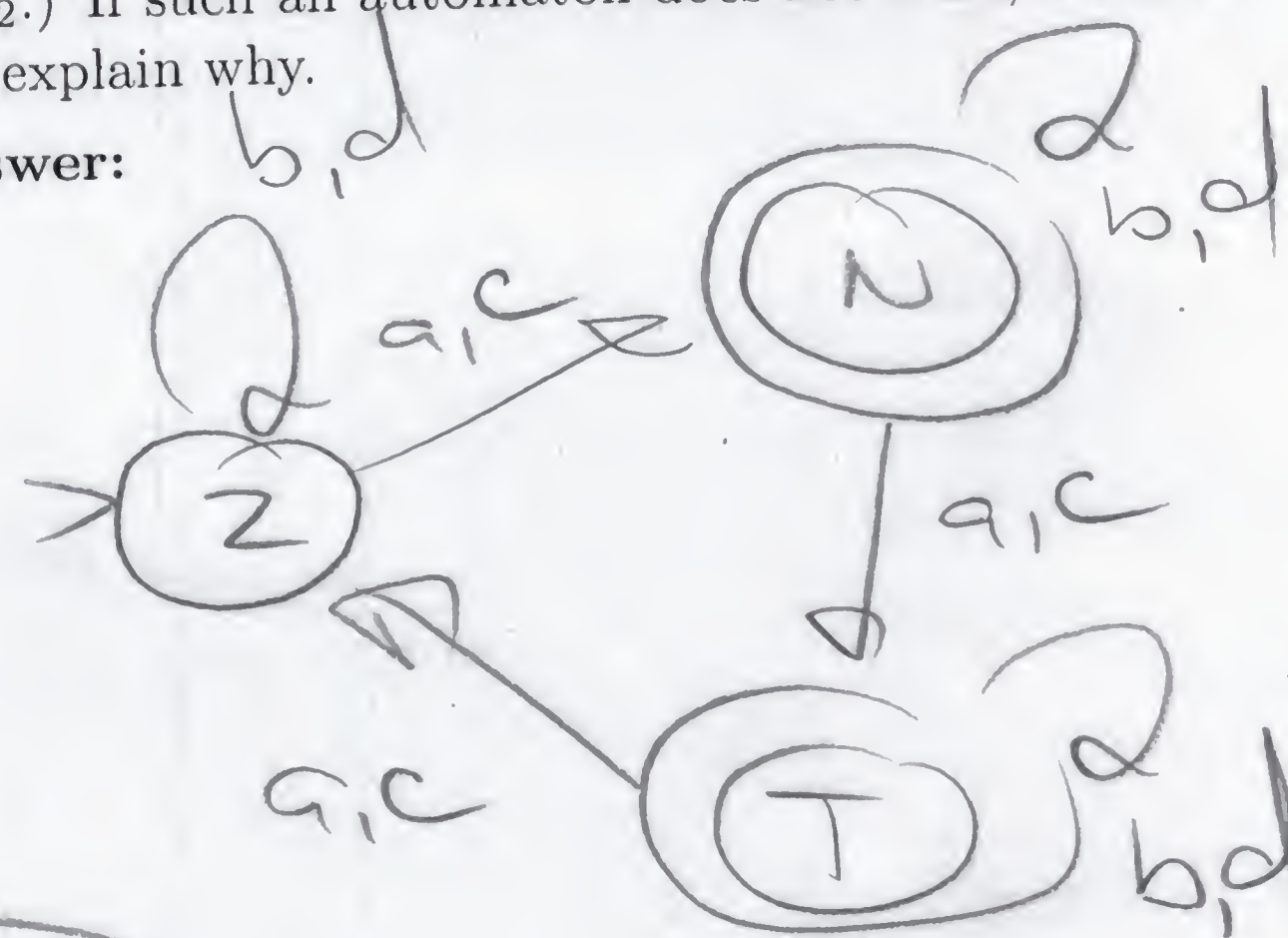
(b) Draw a state-transition graph of a finite automaton that accepts the language L_2 . If such an automaton does not exist, state it and explain why.

Answer:



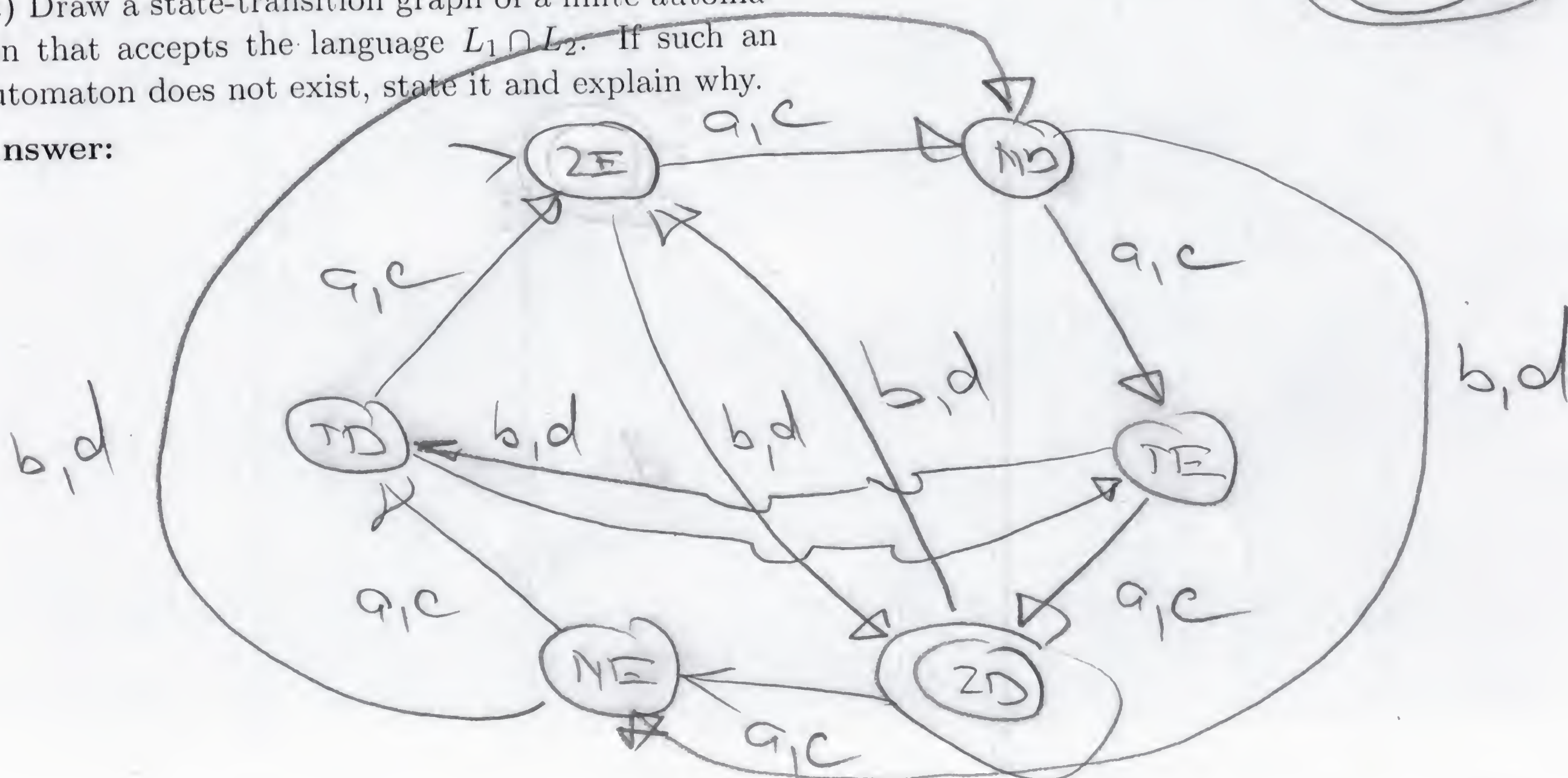
(e) Draw a state-transition graph of a finite automaton that accepts the language $\overline{L_2}$ (the complement of L_2 .) If such an automaton does not exist, state it and explain why.

Answer:



(c) Draw a state-transition graph of a finite automaton that accepts the language $L_1 \cap L_2$. If such an automaton does not exist, state it and explain why.

Answer:



Problem 5 Let L be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties:

1. begins and ends with the same letter;
2. contains exactly two c 's.

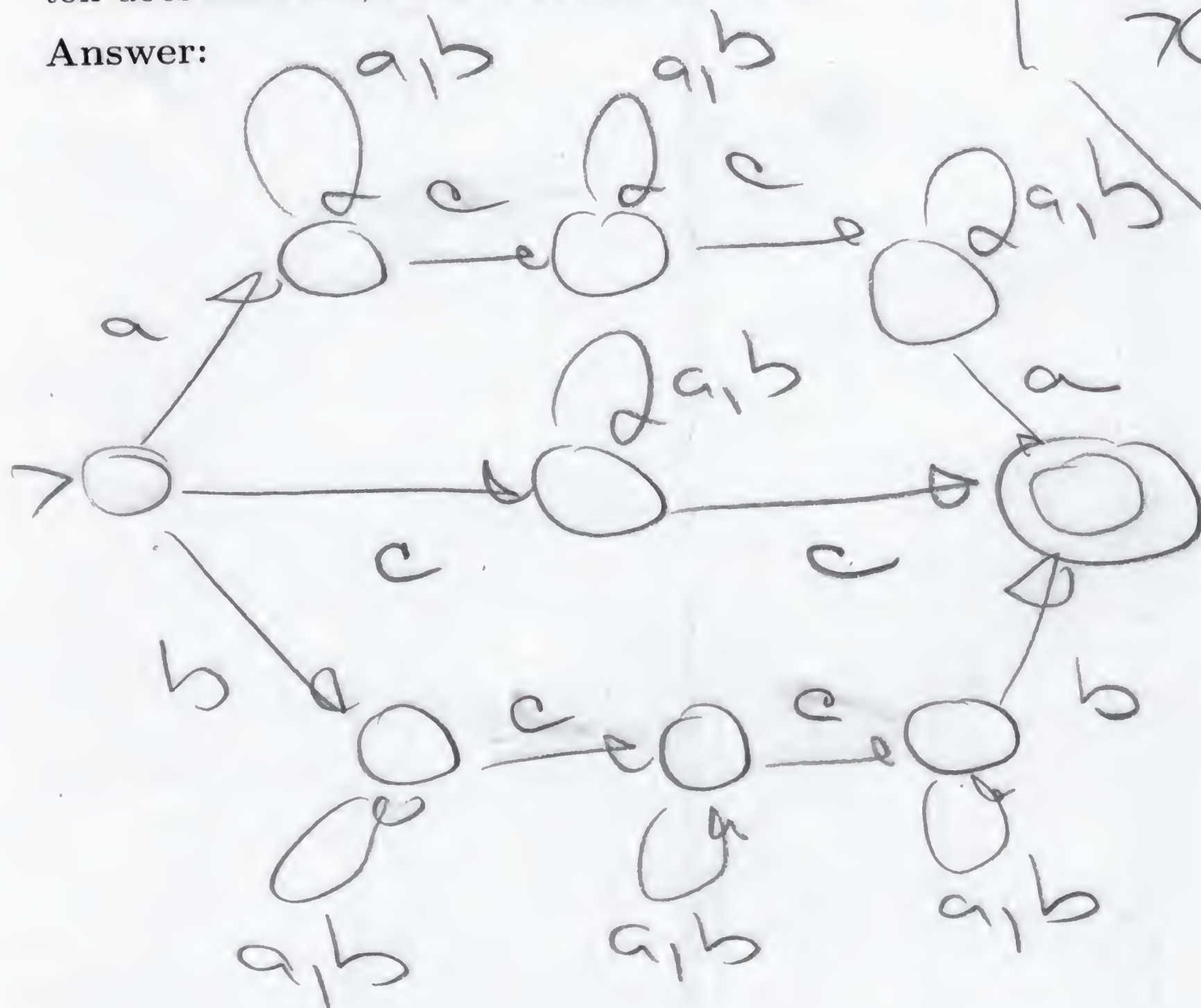
(a) Write a regular expression that represents the language L . If such a regular expression does not exist, state it and explain why.

Answer:

$$a(aub)^*c(aub)^*c(aub)^*a \cup b(aub)^*c(aub)^*c(aub)^*b \cup c(aub)^*c$$

(b) Draw a state-transition graph of a finite automaton that accepts the language L . If such an automaton does not exist, state it and explain why.

Answer:



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(c) Write a complete formal definition of a context-free grammar that generates the language L . If such a grammar does not exist, state it and explain why.

Answer:

$$G = (V, \Sigma, P, S)$$

$$\Sigma = \{a, b, c\}$$

$$V = \{S, A, B, K, D\}$$

$$P: S \rightarrow A | B | K$$

$$A \rightarrow a D c D c D a$$

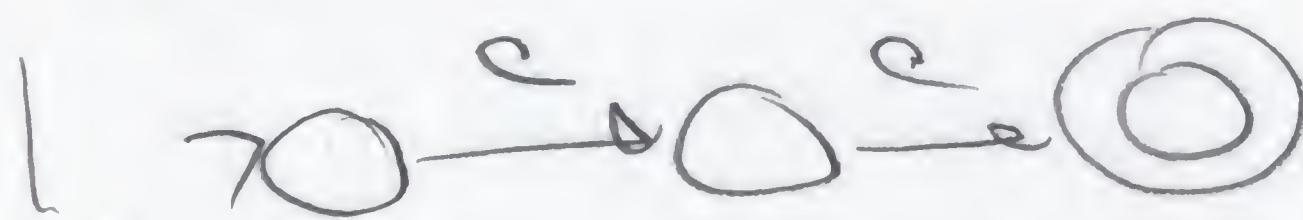
$$B \rightarrow b D c D c D b$$

$$K \rightarrow c D c$$

$$D \rightarrow \epsilon | D D | a | b$$

(d) Draw a state-transition graph of a finite automaton that accepts the language $L \cap c^*$. If such an automaton does not exist, state it and explain why.

Answer:



Problem 5 Let L be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties:

1. first letter is either a or b ;
2. last letter is either b or c ;
3. first letter is different from the last letter;
4. contains exactly two c 's.

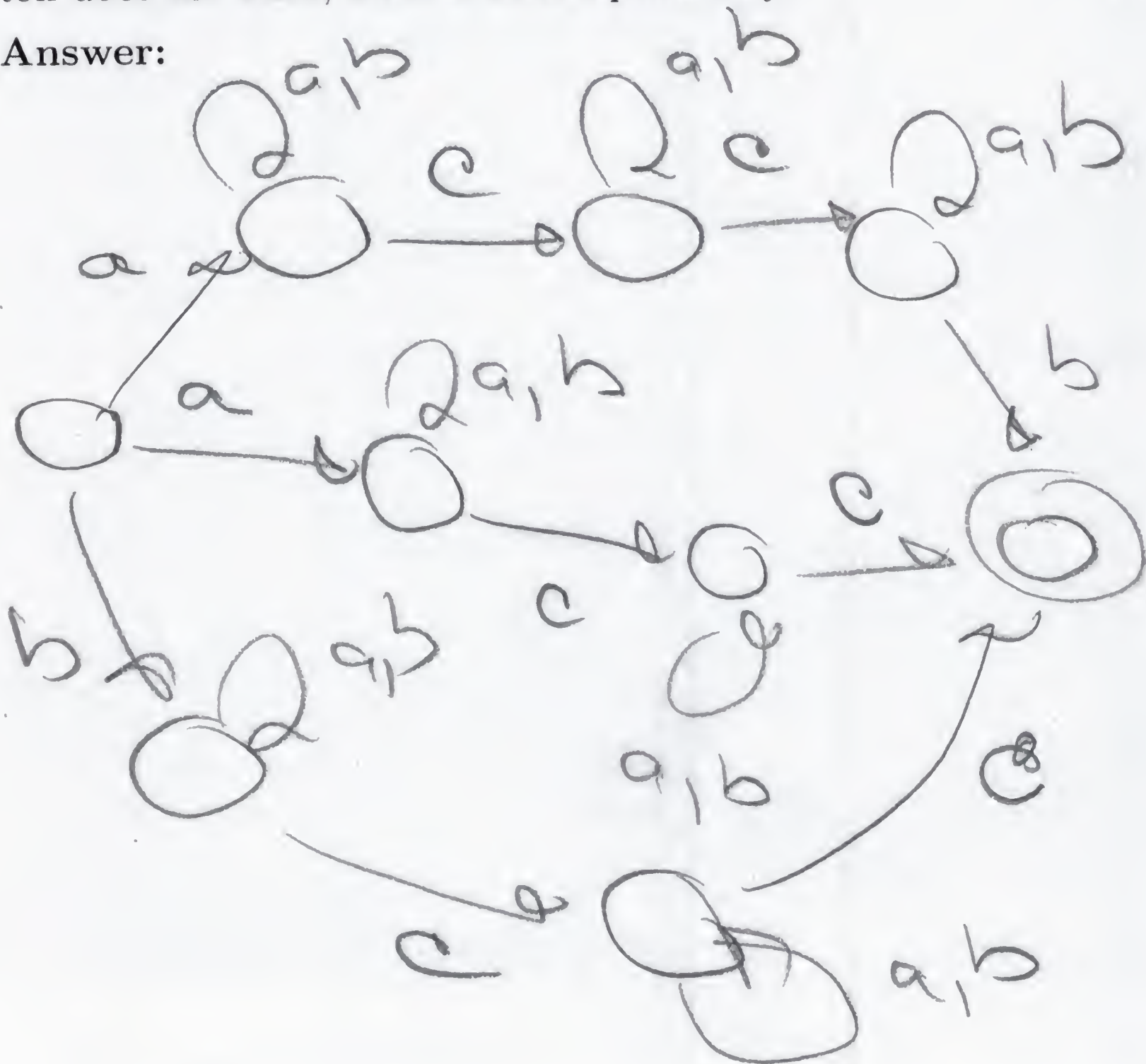
(a) Write a regular expression that represents the language L . If such a regular expression does not exist, state it and explain why.

Answer:

$$a(aub)^*c(aub)^*c(aub)^*b \\ \cup \\ a(aub)^*c(aub)^*c \\ \cup \\ b(aub)^*c(aub)^*c$$

(b) Draw a state-transition graph of a finite automaton that accepts the language L . If such an automaton does not exist, state it and explain why.

Answer:



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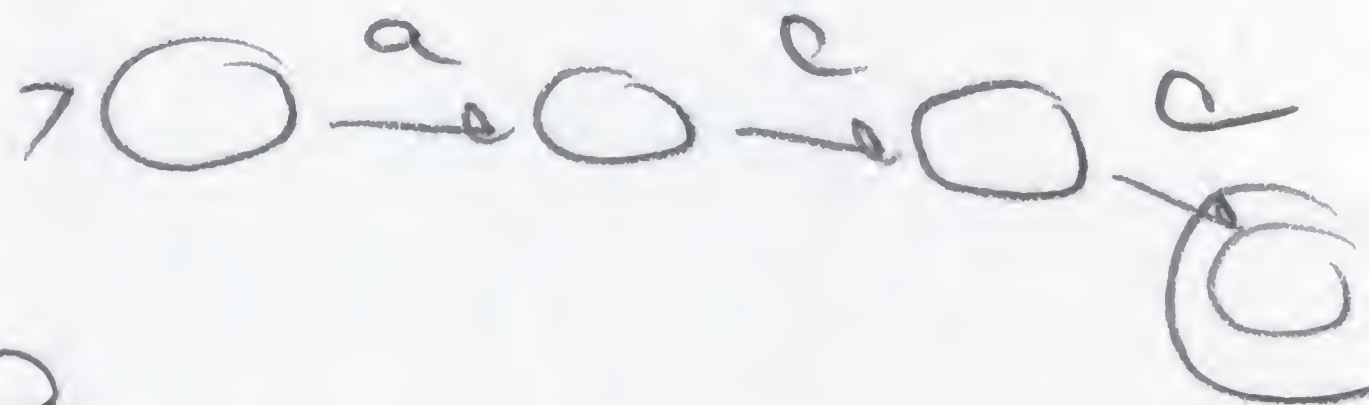
(c) Write a complete formal definition of a context-free grammar that generates the language L . If such a grammar does not exist, state it and explain why.

Answer:

$$G = (V, \Sigma, P, S) \\ \Sigma = \{a, b, c\} \\ V = \{S, A, B, D, E\} \\ S \rightarrow A | B | D \\ A \rightarrow a E c E c E b \\ B \rightarrow a E c E c \\ D \rightarrow b E c E b \\ E \rightarrow \epsilon | E E | a | b$$

(d) Draw a state-transition graph of a finite automaton that accepts the language $L \cap ac^*$. If such an automaton does not exist, state it and explain why.

Answer:



Problem 6 Let L_1, L_2 be languages over the alphabet $\{a, b, c, d, g, e\}$, defined as follows:

$$L_1 = \{g^{3k} e^{2i+3} d^{2\ell} c^{2t+1} b^\ell a^k\}$$

$$L_2 = \{c^{2m+3} a^{3m+1} d^{2n} g^{j+2} e^{3p} b^{j+1}\}$$

where $m, j, n, p, i, k, \ell, t \geq 0$.

(a) Write a complete formal definition of a context-free grammar that generates L_1 . If such a grammar does not exist, state it and explain why.

Answer:

$$\begin{aligned} G &= (V, \Sigma, P, T_1) \\ \Sigma &= \{a, b, c, d, g\} \\ V &= \{T_1, A, B, D\} \\ P: T_1 &\rightarrow ggg T_1 a \mid AB \\ A &\rightarrow eee A \mid eee \\ B &\rightarrow dd B b \mid D \\ D &\rightarrow cc D \mid c \end{aligned}$$

(b) Write a complete formal definition of a context-free grammar that generates L_2 . If such a grammar does not exist, state it and explain why.

Answer:

$$\begin{aligned} G &= (V, \Sigma, P, T_2) \\ V &= \{T_2, E, F, H, J\} \\ \Sigma &= \{a, b, c, d, g\} \\ P: T_2 &\rightarrow E F H \\ E &\rightarrow cc E a a a \mid c c c c \\ F &\rightarrow dd F \mid \lambda \\ H &\rightarrow g H b \mid g g J b \\ J &\rightarrow e e e J \mid \lambda \end{aligned}$$

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(c) Write a complete formal definition of a context-free grammar that generates $(L_1 \cup L_2)^*$. If such a grammar does not exist, state it and explain why.

Answer:

$$\begin{aligned} G &= (V, \Sigma, P, S) \\ V &= \{S, T_1, A, B, D, T_2, E, F, H, J\} \\ \Sigma &= \{a, b, c, d\} \\ P: S &\rightarrow \lambda \mid S S \mid T_1 \mid T_2 \\ T_1 &\rightarrow g g g T_1 a \mid AB \\ A &\rightarrow e e e A \mid e e e \\ B &\rightarrow d d B b \mid D \\ D &\rightarrow c c D \mid c \\ T_2 &\rightarrow E F H \\ E &\rightarrow c c E a a a \mid c c c c \end{aligned}$$

(d) Write a complete formal definition of a context-free grammar that generates $L_1^* \cup L_2^*$. If such a grammar does not exist, state it and explain why.

Answer:

$$\begin{aligned} G &= (V, \Sigma, P, S) \\ \Sigma &= \{a, b, c, d, g\} \\ V &= \{S, S_1, S_2, T_1, T_2, A, B, D, E, F, H, J\} \\ P: S &\rightarrow S_1 \mid S_2 \\ S_1 &\rightarrow \lambda \mid S_1 S_1 \mid T_1 \\ S_2 &\rightarrow \lambda \mid S_2 S_2 \mid T_2 \\ T_1 &\rightarrow g g g T_1 a \mid AB \\ A &\rightarrow e e e A \mid e e e \\ B &\rightarrow d d B b \mid D \\ D &\rightarrow c c D \mid c \\ T_2 &\rightarrow E F H \\ E &\rightarrow c c E a a a \mid c c c c \end{aligned}$$

Problem 6 Let L_1, L_2 be languages over the alphabet $\{a, b, c, d, g, e\}$, defined as follows:

$$L_1 = \{a^{3m+2} c^{2m+1} e^{2n} b^{j+3} g^{3p} d^{j+2}\}$$

$$L_2 = \{b^k g^{2i+1} a^\ell e^{2t+3} d^{2\ell} c^{3k}\}$$

where $m, j, n, p, i, k, \ell, t \geq 0$.

(a) Write a complete formal definition of a context-free grammar that generates L_1 . If such a grammar does not exist, state it and explain why.

Answer:

$$\begin{aligned} G &= (V, \Sigma, P, S) \\ \Sigma &= \{a, b, c, d, g\} \\ V &= \{S, A, B, D, E\} \\ P: \quad &S \rightarrow ABD \\ &A \rightarrow aaaAcc/aac \\ &B \rightarrow eeB/\lambda \\ &D \rightarrow bDd/bbbEdd \\ &E \rightarrow gggE/\lambda \end{aligned}$$

(b) Write a complete formal definition of a context-free grammar that generates L_2 . If such a grammar does not exist, state it and explain why.

Answer:

$$\begin{aligned} G &= (V, \Sigma, P, T) \\ V &= \{T, F, H, \sqsupset\} \\ \Sigma &= \{a, b, c, d, g\} \\ P: \quad &T \rightarrow bTccc/F\# \\ &F \rightarrow gggF/\lambda \\ &H \rightarrow aHdd/\sqsupset \\ &\sqsupset \rightarrow ee\sqsupset/eee \end{aligned}$$

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(c) Write a complete formal definition of a context-free grammar that generates $L_1^* \cup L_2^*$. If such a grammar does not exist, state it and explain why.

Answer:

$$\begin{aligned} G &= (V, \Sigma, P, Q) \\ V &= \{Q, Q_1, Q_2, S, A, B, D, E, T, F, H, \sqsupset\} \\ \Sigma &= \{a, b, c, d, g\} \\ P: \quad &Q \rightarrow Q_1/Q_2 \\ &Q_1 \rightarrow \lambda/Q_1Q_1/S \\ &S \rightarrow ABD \\ &A \rightarrow aaaAcc/aac \\ &B \rightarrow eeB/\lambda \\ &D \rightarrow bDd/bbbEdd \\ &E \rightarrow gggE/\lambda \\ &T \rightarrow bTccc/F\# \end{aligned}$$

(d) Write a complete formal definition of a context-free grammar that generates $(L_1 \cup L_2)^*$. If such a grammar does not exist, state it and explain why.

Answer:

$$\begin{aligned} G &= (V, \Sigma, P, Q) \\ V &= \{Q, S, A, B, D, E, T, F, H, \sqsupset\} \\ \Sigma &= \{a, b, c, d, g\} \\ P: \quad &Q \rightarrow \lambda/QQ/S \\ &S \rightarrow ABD \\ &A \rightarrow aaaAcc/aac \\ &B \rightarrow eeB/\lambda \\ &D \rightarrow bDd/bbbEdd \\ &E \rightarrow gggE/\lambda \\ &T \rightarrow bTccc/F\# \end{aligned}$$

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Problem 7 Let L be the set of exactly those strings over the alphabet $\{a, b, c, d\}$ which satisfy all of the following properties.

1. the string is a concatenation of four non-empty palindromes;
2. three of the four palindromes have an odd length;
3. one of the four palindromes has an even length;
4. the four palindromes may appear in any order;
5. the middle symbol of each of the three odd-length palindromes is different from d ;
6. the middle two symbols of the even-length palindrome are different from a .

Write a complete formal definition of a context-free grammar that generates the language L . If such a grammar does not exist, state it and explain why.

Answer:

$$G = (V, \Sigma, P, S)$$

$$\Sigma = \{a, b, c, d\}$$

$$V = \{S, E, D\}$$

$$P: \begin{aligned} S &\rightarrow EDDDD / DEDDD / DDDED / DDDDE \\ E &\rightarrow aEa / bEb / cEc / dEd / bb / cc / dd \\ D &\rightarrow aDa / bDb / cDc / a / b / c \end{aligned}$$

Problem 7 Let L be the set of exactly those strings over the alphabet $\{a, b, c\}$ which satisfy all of the following properties.

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FIRST NAME: _____

1. the string is a concatenation of four non-empty palindromes;
2. three of the four palindromes have an even length;
3. one of the four palindromes has an odd length;
4. the four palindromes may appear in any order;
5. the middle symbol of the odd-length palindrome is different from a ;
6. the middle two symbols of each of the three even-length palindromes are different from d .

Write a complete formal definition of a context-free grammar that generates the language L . If such a grammar does not exist, state it and explain why.

Answer:

$$G = (V, \Sigma, P, S)$$

$$\Sigma = \{a, b, c\}$$

$$V = \{S, E, D\}$$

$$P : S \rightarrow DEEE \mid EDEE \mid EEDE \mid EEEED$$

$$E \rightarrow aEa \mid bEb \mid cEc \mid aa \mid bb \mid cc$$

$$D \rightarrow aDa \mid bDb \mid cDc \mid b \mid c$$